Application of the Two-Scale Model to the HERMES Data on Nuclear Attenuation

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Abstract

The Two-Scale Model and its improved version were used to perform the fit to the HERMES data for ν (the virtual photon energy) and z (the fraction of ν carried by hadron) dependencies of nuclear multiplicity ratios for π^+ and π^- mesons electro-produced on two nuclear targets (^{14}N and ^{84}Kr). The quantitative criterium χ^2 was used for the first time to analyse the results of the model fit to the nuclear multiplicity ratios data. The two-parameter's fit gives satisfactory agreement with the HERMES data. Best values of the parameters were then used to calculate the ν- and z - dependencies of nuclear attenuation for π^0, K$^+$, K$^-$ and $\bar{p}$ produced on ^{84}Kr target, and also make a predictions for ν, z and the Q^2 (the photon virtuality) - dependencies of nuclear attenuation data for those identified hadrons and nuclea, that will be published by HERMES.

1 Introduction

Studies of hadron production in deep inelastic semi-inclusive lepton-nucleus scattering (SIDIS) offer a possibility to investigate the quark (string, color dipole) propagation in dense nuclear matter and the space-time evolution of the hadronization process. It is well-known from QCD, that confinement forbids existence of an isolated color charge (quark, antiquark, etc.). Consequently, it is clear that after Deep Inelastic Scattering (DIS) of lepton on intra-nuclear nucleon, the complicated colorless pre-hadronic system arises. Its propagation in the nuclear environment involves processes like multiple interactions with the surrounding medium and induced gluon radiation. If the final hadron is formed inside the nucleus, the hadron can interact via the relevant hadronic cross section, causing further reduction of the hadron yield [1]. QCD at present can not describe the process of quark hadronization because of the major role of "soft" interactions. Therefore, the investigation of quark hadronization is of basic importance for development of QCD. For this purpose we investigate in this paper the Nuclear Attenuation (NA), which is the ratio of the differential multiplicity on nucleus to that on deuterium. At present there exist numerous phenomenological models for investigation of the NA problem [2]-[14]. In this work we use the Two-Scale Model [4] and its improved version to perform the fit to the HERMES NA data [15, 16]. For the fitting purposes we use the more precise part of data, including data for ν- and z - dependencies of NA of π^+ and π^- mesons on two nuclear targets (^{14}N and ^{84}Kr). The ν- and z - dependencies of NA for π^0, K$^+$, K$^-$ and antiproton, produced on

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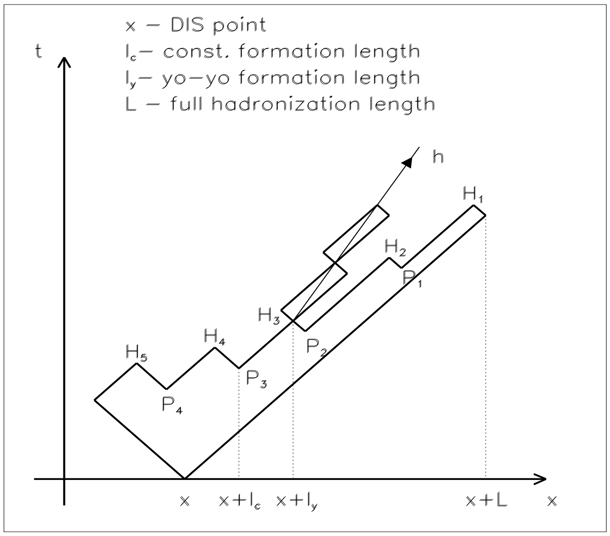

Figure 1: Space-time structure of hadronization in the string model. The two constituents of the hadron are produced at different points. The constituents of the hadron h are created at the points P_2 and P_3. They meet at H_3 to form the hadron.

^{84}Kr target we describe with best values of parameters obtained from the above mentioned fit. The best set of parameters are used also for prediction of ν, z and Q^2- dependencies of NA for the data on those identified hadrons and targets that will be published soon by HERMES [17]. The remainder of the paper is organized as follows. In section 2 we briefly remind about the Two-Scale Model. In section 3 we discuss the possibility of inclusion of the Q^2-dependence in the Two-Scale Model . In section 4 we describe the scheme we used to improve the Two-Scale Model, substituting the step-by-step increase of the string-nucleon cross section by a smooth raising function. In section 5 we present the results of the model fit to the HERMES data. Our conclusions are given in section 6.

2 The Two-Scale Model

The Two-Scale Model is a string model, which was proposed by EMC [4] and used for the description of their experimental data. Basic formula is:

$$R_A = 2\pi \int_0^\infty bdb \int_{-\infty}^\infty dx \rho(b,x)[1 - \int_x^\infty dx' \sigma^{str}(\Delta x)\rho(b,x')]^{A-1} \quad (1)$$

where b - impact parameter, x - longitudinal coordinate of the DIS point, x′- longitudinal coordinate of the string-nucleon interaction point, $\sigma^{str}(\Delta x)$ - the string-nucleon cross section on distance Δx = x'-x from DIS point, $\rho(b,x)$ - nuclear density function, A - atomic mass number.

The model contains two scale (see Fig. 1): τ_c (l_c) - constituent formation time (length), and τ_h (l_h) - yo-yo formation time (length), [2] yo-yo formation means, that the colorless

[2]in relativistic units ($\hbar$ = c = 1, where $\hbar = h/2\pi$ is the Plank reduced constant and c - speed of light)

system with valence content and quantum numbers of final hadron arises, but without its "sea" partons. The simple connection exists between τ_h and τ_c

$$\tau_h - \tau_c = z\nu/\kappa, \tag{2}$$

where z= E_h/ν, E_h and ν are energies of final hadron and virtual photon correspondingly, κ - string tension (string constant). Further we will use two different expressions for τ_c. The expression for τ_c obtained for hadrons containing leading quark [18]:

$$\tau_c = (1 - z)\nu/\kappa. \tag{3}$$

The expression for average value of τ_c, which was obtained in [5, 19] in framework of the standard Lund model [20]:

$$\tau_c = \int_0^\infty l dl D_c(L, z, l) / \int_0^\infty dl D_c(L, z, l), \tag{4}$$

where $D_c(L, z, l)$ is the distribution of the constituent formation length l of hadrons carrying momentum z. This distribution is:

$$D_c(L, z, l) = L(1 + C)\frac{l^C}{(l + zL)^{C+1}}(\delta(l - L + zL) + \frac{1 + C}{l + zL})\theta(l)\theta(L - zL - l), \tag{5}$$

where L = ν/κ, and parameter C=0.3. The path traveling by string between DIS and interaction points is Δx = x'-x. The string-nucleon cross section is:

$$\sigma^{str}(\Delta x) = \theta(\tau_c - \Delta x)\sigma_q + \theta(\tau_h - \Delta x)\theta(\Delta x - \tau_c)\sigma_s + \theta(\Delta x - \tau_h)\sigma_h \tag{6}$$

where σ_q, σ_s and σ_h are the cross sections for interaction with nucleon of initial string, open string (which becomes one of the hadron quarks being looked at) and final hadron respectively (see Fig.2 a)).

3 Inclusion of the Q^2-dependence in Two-Scale Model.

The Two-Scale Model [4] does not contain direct Q^2-dependence and operates with the average values of cross sections:

$$\sigma_q = \sigma_q(\hat{Q}^2); \qquad \sigma_s = \sigma_s(\hat{Q}^2_{\tau_c}), \tag{7}$$

where $\hat{Q}^2$ is average value of Q^2 obtained in experiment for initial state and $\hat{Q}^2_{\tau_c}$ is the same for open string, or for time τ_c after DIS point. Obviously that $\hat{Q}^2_{\tau_c}$ is smaller than $\hat{Q}^2$.

because after DIS the string radiates gluons and diminishes its virtuality. QCD predicts the Q^2-dependence of string-nucleon cross section in the form [21, 22]:

$$\sigma_q(Q^2) \sim 1/Q^2; \qquad \sigma_s(Q^2_{\tau_c}) \sim 1/Q^2_{\tau_c}. \tag{8}$$

Using this prediction we can write the cross section for initial string as

$$\sigma_q(Q^2) = (\hat{Q}^2/Q^2)\sigma_q(\hat{Q}^2). \tag{9}$$

In the same way can be written the expression for open string cross section

$$\sigma_s(Q^2_{\tau_c}) = (\hat{Q}^2_{\tau_c}/Q^2_{\tau_c})\sigma_s(\hat{Q}^2_{\tau_c}), \tag{10}$$

where $Q^2_{\tau_c}$ is the virtuality of string for time τ_c after DIS point, and $\hat{Q}^2_{\tau_c}$ is the same for average value of Q^2. For estimation of ratio $\hat{Q}^2_{\tau_c}/Q^2_{\tau_c}$ we adopt the scheme given in Ref. [23, 24]. In according with this scheme, for the time t the quark decreases its virtuality from the initial one, Q^2, to the value $Q^2(t)$

$$Q^2(t) = \nu(t)\frac{Q^2}{\nu(t) + tQ^2}, \tag{11}$$

where $\nu(t) = \nu - \kappa t$. The calculations shown, that for HERMES kinematics ($1.2 < Q^2 < 9.5$ GeV^2 and $\hat{Q}^2 = 2.5$ GeV^2), values for ratio $\hat{Q}^2_{\tau_c}/Q^2_{\tau_c}$ are close to 1 (for τ_c in form of (3) it changes in region $0.97 \div 1.04$ and in form of (4) in region $0.92 \div 1.12$). This means that σ_s is practically constant.

4 Improved version of Two-Scale Model

In the Two-Scale Model the string-nucleon cross section is a function which jumps in points $\Delta x = \tau_c$ and τ_h. In reality the cross section increases smoothly until it reaches the size of hadronic cross section. That is why we need to improve the model in order to obtain the smooth increase of the cross section (see Fig. 2). We introduce the parameter c ($0 < c < 1$) in order to take into account the well known fact, that string starts to interact with hadronic cross section soon after creation of the first constituent quark of the final hadron, before creation of second constituent. The string-nucleon cross section starts to increase from DIS point, and reaches the value of the hadron-nucleon cross section at $\Delta x = \tau$. However, in that case one cannot deduce the exact form of σ^{str} from perturbative QCD, at least in region $\Delta x \sim \tau$. This means, that some model for the shrinkage-expansion mechanism has to be invented. We use four versions for σ^{str}. Two versions we took from Ref. [25]. The first version is based on quantum diffusion:

$$\sigma^{str}(\Delta x) = \theta(\tau - \Delta x)[\sigma_q + (\sigma_h - \sigma_q)\Delta x/\tau] + \theta(\Delta x - \tau)\sigma_h \tag{12}$$

where $\tau = \tau_c + c\Delta\tau$, $\Delta\tau = \tau_h - \tau_c$,

The second version follows from naive parton case:

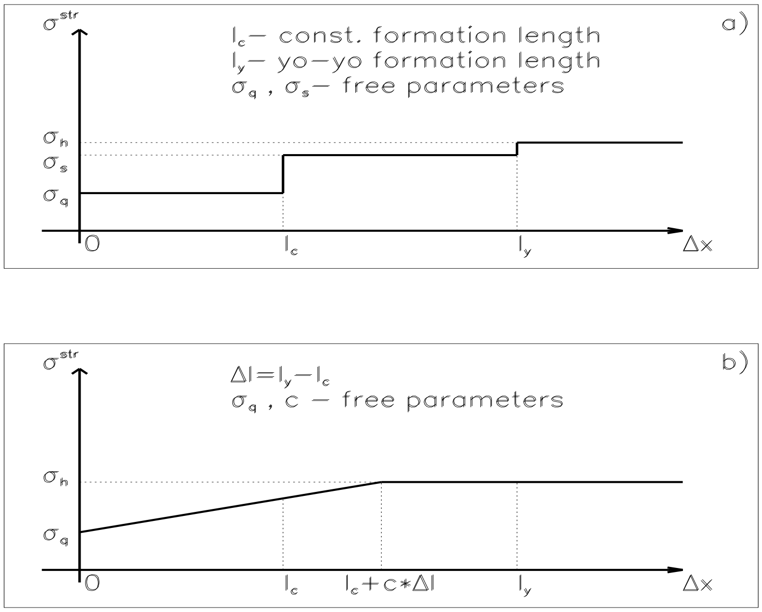

Figure 2: a) The behaviour of the string-nucleon cross section as a function of distance in the Two-Scale Model. b) The same as in a) for improved Two-Scale Model with taking into account more realistic smoothly increasing string-nucleon cross section.

$$\sigma^{str}(\Delta x) = \theta(\tau - \Delta x)[\sigma_q + (\sigma_h - \sigma_q)(\Delta x/\tau)^2] + \theta(\Delta x - \tau)\sigma_h \tag{13}$$

We used also two other expressions for σ^{str} [2, 6]:

$$\sigma^{str}(\Delta x) = \sigma_h - (\sigma_h - \sigma_q)exp(-\Delta x/\tau) \tag{14}$$

and:

$$\sigma^{str}(\Delta x) = \sigma_h - (\sigma_h - \sigma_q)exp(-(\Delta x/\tau)^2) \tag{15}$$

One can easily note that at $\Delta x/\tau \ll 1$ the expressions (14) and (15) turn into (12) and (13), correspondingly.

5 Results

In this work we have formulated one of possible improvements of the Two-Scale model and performed a fit to the HERMES data [15, 16]. Only the data for NA for ν- and z - dependencies of π^+ and π^- mesons on ^{14}N and ^{84}Kr nuclea were used for the actual fit. Furthermore, NA for ν- and z - dependencies of other hadrons produced on ^{84}Kr target were calculated. Also based on the best fit parameters one can make different predictions for ν, z and Q^2 - dependencies for these identified hadrons and nuclea, that will be published by

HERMES [17]. The string tension (string constant) was fixed at a static value determined by the Regge trajectory slope [24, 26]

$$\kappa = 1/(2\pi\alpha'_R) = 1GeV/fm \tag{16}$$

We use the following Nuclear Density Functions (NDF):

For ^{4}He and ^{14}N we use the Shell Model [27], according to which four nucleons (two protons and two neutrons), fill the s - shell, and other A-4 nucleons are on the p - shell:

$$\rho(r) = \rho_0(\frac{4}{A} + \frac{2}{3}\frac{(A-4)}{A}\frac{r^2}{r_A^2})exp(-\frac{r^2}{r_A^2}), \tag{17}$$

where r_A=1.31 fm for ^{4}He and r_A=1.67 fm for ^{14}N.

For ^{20}Ne, ^{84}Kr and ^{131}Xe we use Woods-Saxon distribution

$$\rho(r) = \rho_0/(1 + exp((r - r_A)/a)). \tag{18}$$

The three sets of NDF were used for the fitting with the following corresponding parameters:

First set (NDF=1) [28].

$$a = 0.54\ fm; \qquad r_A = (0.978 + 0.0206A^{1/3})A^{1/3}\ fm \tag{19}$$

Second set (NDF=2) [29]

$$a = 0.54\ fm; \qquad r_A = (1.19A^{1/3} - 1.61/A^{1/3})\ fm \tag{20}$$

Third set (NDF=3) [30]

$$a = 0.545\ fm; \qquad r_A = 1.14A^{1/3}\ fm. \tag{21}$$

where ρ_0 are determined from normalization condition:

$$\int d^3r\rho(r) = 1 \tag{22}$$

Parameter a is practically the same for all three sets, radius r_A for the third set is larger approximately on 6% than for second and first sets. From the fit we determined two parameters for Two-Scale Model σ_q and σ_s. In case of the improved Two-Scale Model the fitting parameters are σ_q and c.

Determination of the parameter c is represented in Section 4. For fitting we used two expressions for τ_c, which are equations (3) and (4), and five expressions for $\sigma^{str}(\Delta x)$ (6), (12)- (15). The results of the performed fit are presented in Tables 1, 2a and 2b. As we have mentioned above only part of HERMES experimental data was used for the fitting procedure, including ν - and z - dependencies of NA for π^+ and π^- on ^{14}N and ^{84}Kr nuclea. For each measured bin the information on the values of $\hat{z}$ (averaged over the given ν bin) in case of ν dependence, and $\hat{\nu}$ in case of z dependence was taken from experimental data. Use of this information allows to avoid the problem of additional integration over z and ν in formulae (1).

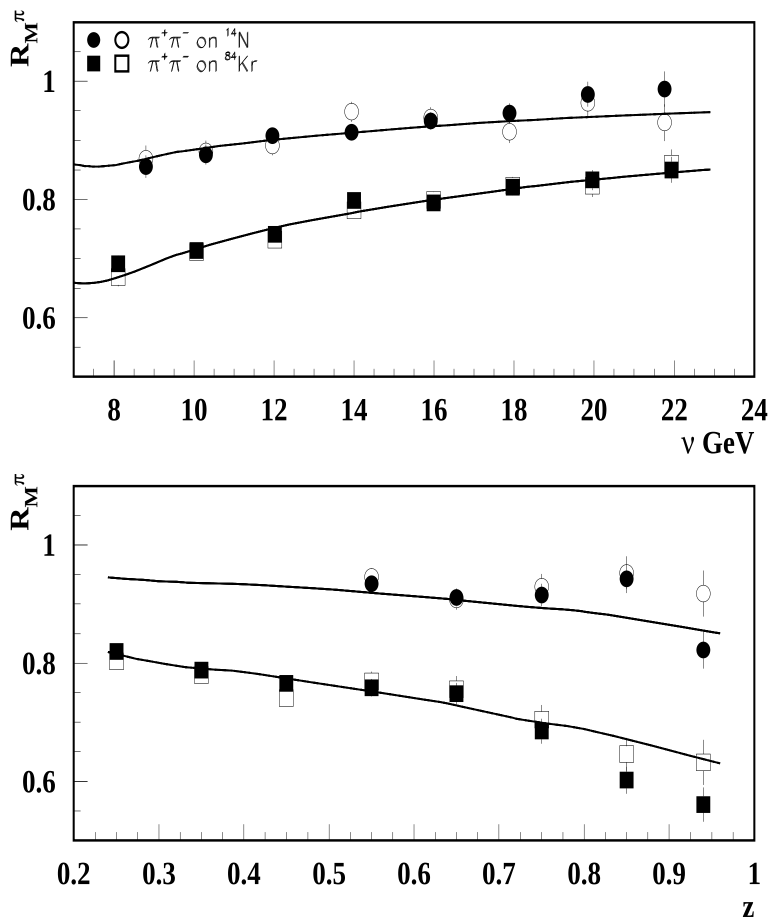

Figure 3: Hadron multiplicity ratio R of charged pions for ^{14}N and ^{84}Kr nuclea as a function of ν (upper panel), z (lower panel). The theoretical curves correspond the calculations in Improved Two-Scale Model performed with NDF (17) for ^{14}N and NDF (19) for ^{84}Kr and σ^{str} (12) with τ_c in form (3) for the values of parameters: σ_q=0.46mb, c=0.32.

In Table 1 the best values for fitted parameters, their errors and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2. N$_{exp}$ and N$_{par}$ are the numbers of experimental points and fitting parameters which were used.) for the Two-Scale Model are represented. Two different expressions for τ_c, and three different sets of parameters for NDF (^{84}Kr) were used. Tables 2a and 2b contain the best values for fitted parameters, their errors and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2) for the Improved Two-Scale Model. Four different expressions for σ^{str} were used. Only difference between Tables 2a and 2b is the form of τ_c. The results for Two-Scale Model (Table 1) are qualitatively close to the results of Ref. [4]. The values of $\sigma_q \ll \sigma_h$ and σ_s are approximately equal to σ_h. σ_q in our case is larger than the same in Ref. [4], because $\hat{Q}^2$ for HERMES kinematics is smaller than in EMC kinematics. The minimum values for χ^2/d.o.f. (best fit) were obtained for the Improved Two-Scale Model with the constituent formation time τ_c in form of (3) (see Table 2a).

The results for NA, calculated with the best values of fitting parameters for improved Two-Scale Model, for ν and z dependencies of produced charged pions on ^{14}N and ^{84}Kr targets are presented on Fig. 3.

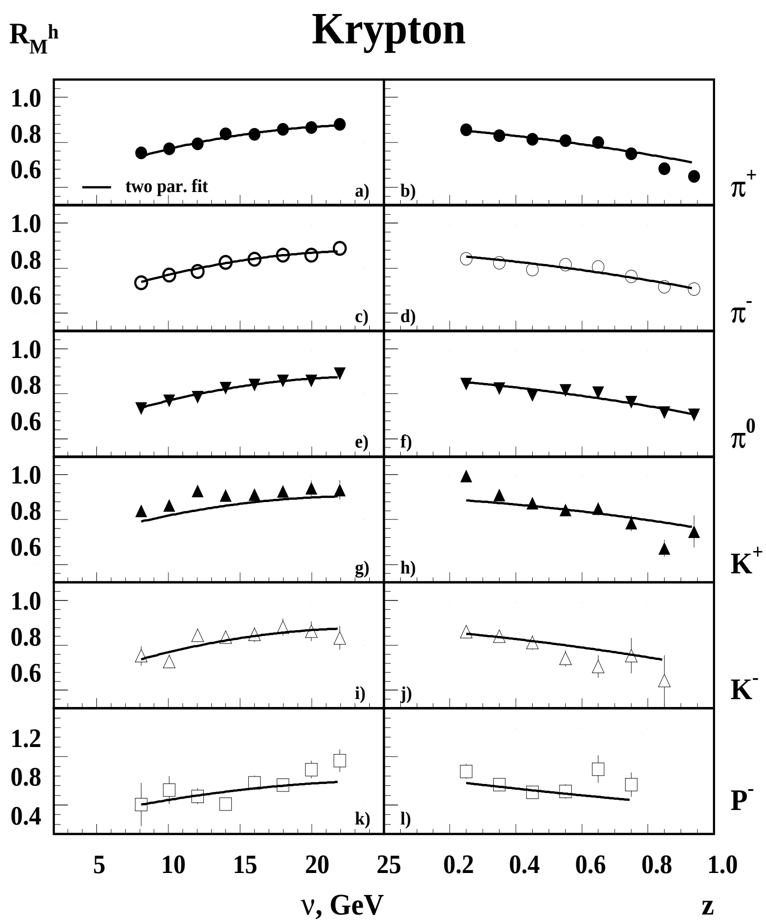

Figure 4: Hadron multiplicity ratio R of different species of hadrons produced on ^{84}Kr target [16] as a function of ν (left panel) and z (right panel). The curves are calculated with the best fit parameters described in the caption of Fig. 3.

In Fig. 4 one can see the ν and z dependencies for all identified hadrons produced on ^{84}Kr target. The values of σ_h (hadron-nucleon inelastic cross section) used in this work are equal to: $\sigma_{\pi^+} = \sigma_{\pi^-} = \sigma_{\pi^0} = \sigma_{K^-} = 20$ mb, $\sigma_{K^+} = 14$ mb and $\sigma_{\bar{p}} = 42$ mb. The curves correspond to the improved Two-Scale model with the best set of parameters.

In Fig. 5 we present the results of Improved Two-Scale Model in comparison with the experimental data for NA of charged hadrons on ^{63}Cu target [4] performed in region of ν and Q^2 values higher, than in HERMES kinematics. In order to compare with the EMC data we redefined σ_q to the $\hat{Q}^2_{EMC}$, according to the expression (9).

We represent the NA ratio as a function of inverse Q^2, because of connection of this dependence with the Higher Twist effects. Indeed, from the equations (9),(6), (12)-(15) and (1) we can conclude, that in first approximation the expansion over the degrees of $1/Q^2$ for NA ratio can be represented in form $R_A = a + b/Q^2$, where b is negative.

One has to note, that for calculation of $1/Q^2$-dependence, the $\sigma_q(Q^2)$ was used instead of σ_q. Corresponding expression is given by (9). We also take into account nuclear effects in deuterium. This means, that instead of a simple formula (1), we use for calculations the ratio of (1) for nucleus to the (1) for deuterium. For deuterium as NDF we use Hard Core

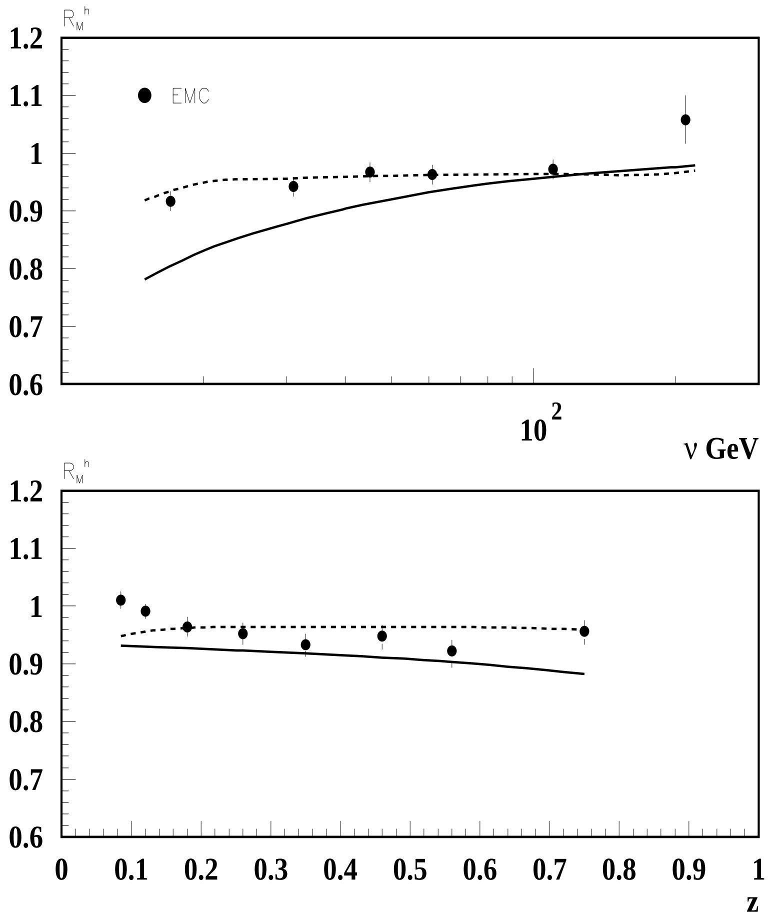

Figure 5: Hadron multiplicity ratio R of charged hadrons for ^{63}Cu nucleus as a function of ν (upper panel) and z (lower panel). The curves are calculated with the best set of parameters described in the caption of Fig. 3

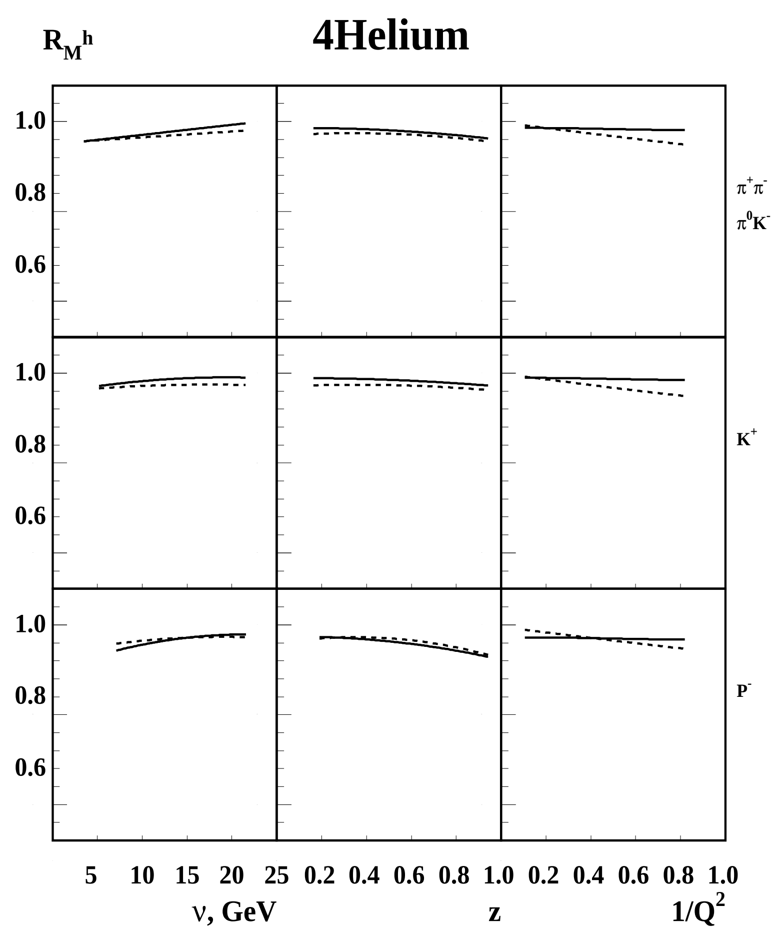

Figure 6: Hadron multiplicity ratio R of different species of hadrons produced on ^{4}He target as a function of ν (left panel), z (central panel) and Q^2 (right panel). The red curves are the best fit using improved Two-Scale model with the parameters described in the caption of Fig. 3. The black curves correspond to the best fit using the simple Two-Scale model with τ_c defined in (4), NDF (17), σ_q = 4.2 mb and σ_s = 16.6 mb

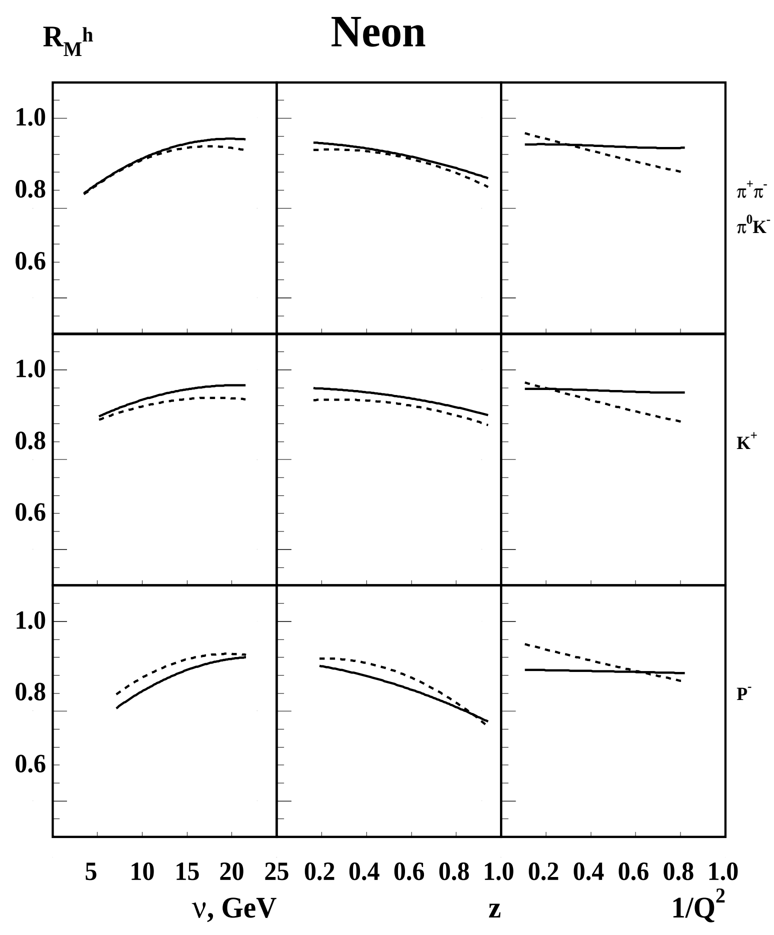

Figure 7: The same as described in the caption of the Fig. 5 done for ^{20}Ne target.

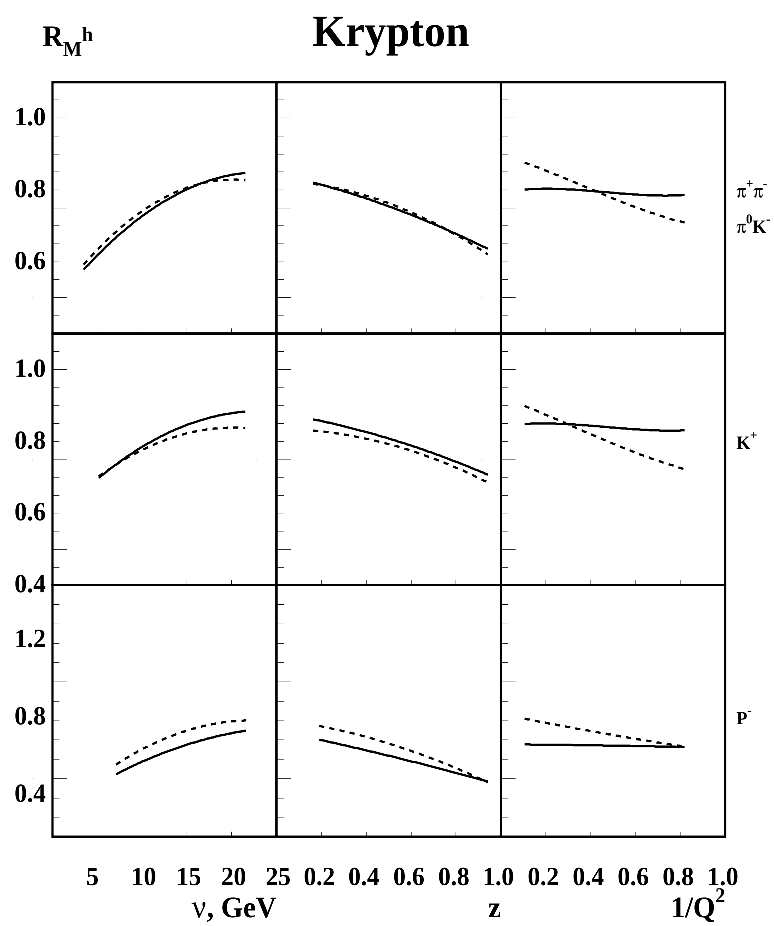

Figure 8: The same as described in the caption of the Fig. 5 done for ^{84}Kr target.

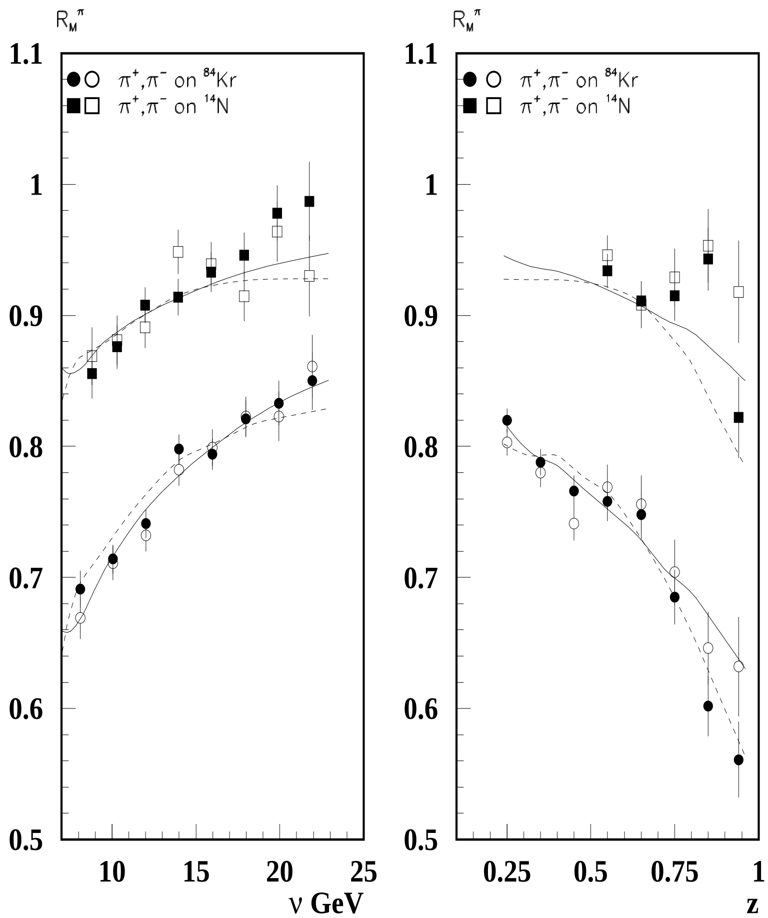

Figure 9: Descriptive ability of the Two-Scale model and its Improved version: right panel - for z, and left panel - for ν dependencies of NA. The red lines on both panels correspond to the Improved version, the blue ones are for simple Two-Scale model.

Deuteron Wave Functions from Ref. [31].

Using the best set of parameters obtained by fitting the published HERMES data [15, 16] we calculated the predictions for the new set of the most precise in the world HERMES data [17] for ^{4}He (Fig. 5),^{20}Ne (Fig. 6) and ^{84}Kr (Fig. 7).

In order to demonstrate the achieved advantages for Imroved Two-Scale model not only on the level of obtained χ^2 values, one can compare how these two versions are describing the NA data for pions on two nuclear targets for z (see right panel of Fig. 8) and ν (see left panel of Fig. 8) dependencies. It's clearly seen from this plot that being about the same for ν dependence these two versions remarkable differ for z dependence.

The last Figure 9 is related to the predictions, done for already presented by HERMES [17] data on ^{4}He, ^{20}Ne and ^{84}Kr targets with the extended kinematics, as well as for ^{131}Xe target, on which the data is awaiting soon from the HERMES Collaboration. Two set of the best fit parameters were fixed: one marked as a blue curves on Fig. 9 is related to the simple Two-Scale Model, next one, marked as a red curves is related to the Improved version of the Two-Scale Model. Left panel corresponds to z dependence of NA for pions, right panel is related to the ν dependence of NA for pions. We can note that again as for

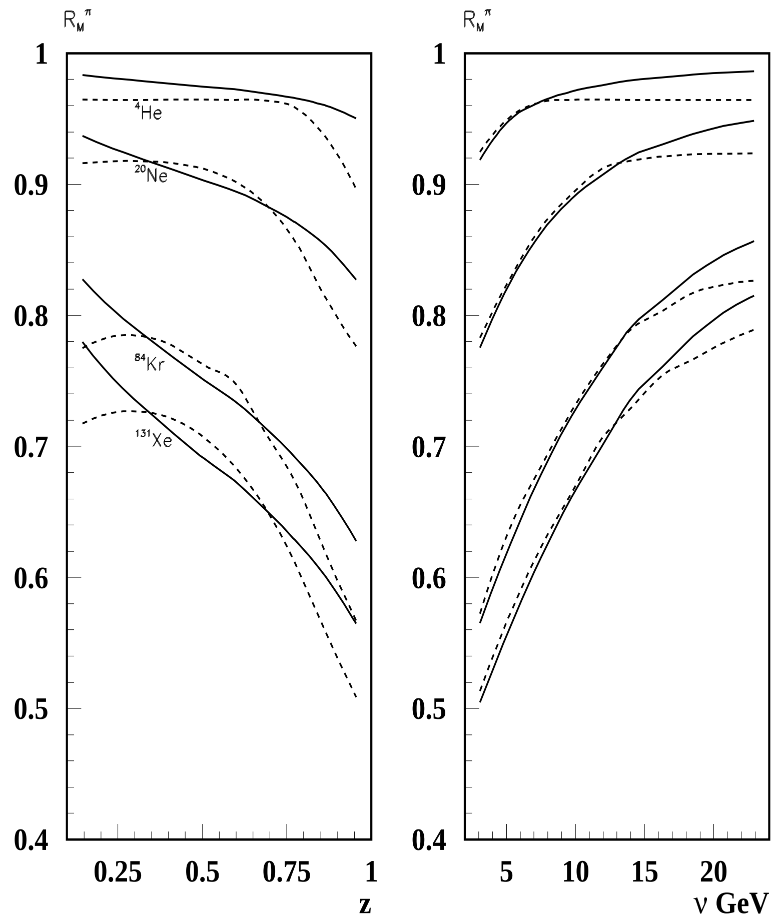

Figure 10: Two-Scale model (blue lines) and its improved version (red lines). The predictions of data on ^{4}He, ^{20}Ne, ^{84}Kr and ^{131}Xe done for z and ν dependencies of NA.

other nuclear targets, the difference in simple and improved versions is remarkable for Xe in z dependence. *

6 Conclusions.

- The HERMES data for ν- and z - dependencies of nuclear attenuation of π^+ and π^- mesons on two nuclear targets (^{14}N and ^{84}Kr) were used to perform the fit of the Two-Scale Model and its Improved Version.
- Criterion χ^2 was used for the first time to analyse the nuclear attenuation data fit.
- Two-parameter fit demonstrates satisfactory agreement to the HERMES data. Minimum χ^2 (best fit) was obtained for improved Two-Scale Model, including expressions (12) for σ^{str} and (3) for τ_c. The published HERMES data do not give the possibility to make a choice between expressions (12)-(15), as well as to prefere definition (3) or (4) for τ_c, because they give close values of χ^2. Preferable NDF's are set number one and two.

- More precise data expected from HERMES [17] will provide essentially definite situation with the choice of preferable NDF, expressions for σ^{str} and τ_c.
- In all versions we have obtained $\sigma_q \ll \sigma_h$. This indicates that at early stage of hadronization process Color Transparency takes place.

	$\tau_c(3)$			$\tau_c(4)$		
NDF	σ_q (mb)	σ_s (mb)	χ^2 /d.o.f.	σ_q (mb)	σ_s (mb)	χ^2 /d.o.f.
1	5.3±0.01	17.1±0.08	4.3	4.2±0.01	16.6±0.07	2.3
2	5.5±0.01	17.7±0.08	4.5	4.3±0.01	17.3±0.07	2.4
3	5.8±0.010	18.3±0.08	4.8	4.4±0.01	18.1±0.07	2.6

Table 1. The Two-Scale Model. Best values for fitted parameters and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2)

	$\sigma_{str}(12)$			$\sigma_{str}(13)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	0.46±0.02	0.32±0.03	1.4	3.5±0.01	0.23±0.002	1.9
2	0.62±0.01	0.31±0.01	1.7	3.7±0.01	0.22±0.02	2.1
3	0.78±0.02	0.30±0.03	1.8	3.9±0.01	0.21±0.003	2.3

	$\sigma_{str}(14)$			$\sigma_{str}(15)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	1.1±0.01	0.15±0.03	2.1	3.7±0.01	0.15±0.02	2.3
2	1.3±0.02	0.15±0.03	2.4	3.9±0.01	0.14±0.02	2.6
3	1.5±0.02	0.14±0.03	2.8	4.1±0.01	0.14±0.02	2.9

Table 2a. The Improved Two-Scale Model:$\tau_c(3)$. Best values for fitted parameters and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2).

	$\sigma_{str}(12)$			$\sigma_{str}(13)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	0.0±0.001	0.56±0.02	4.6	0.97±0.01	0.17±0.002	1.6
2	0.0±0.002	0.53±0.02	4.3	1.0±0.02	0.17±0.02	1.5
3	0.0±0.002	0.49±0.006	4.0	1.1±0.02	0.16±0.02	1.6

	$\sigma_{str}(14)$			$\sigma_{str}(15)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	0.0±0.001	0.24±0.02	3.0	1.5±0.02	0.103±0.02	1.5
2	0.0±0.002	0.21±0.02	2.9	1.7±0.02	0.096±0.02	1.6
3	0.0±0.002	0.18±0.02	2.8	1.8±0.02	0.089±0.02	1.8

Table 2b. The Improved Two-Scale Model:$\tau_c(4)$. Best values for fitted parameters and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2).

We do not include in consideration NA of protons, because in this case additional mechanisms connected with color interaction (string- flip) and final hadron rescattering become essential (see for instance Ref. [3, 5])

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Application of the Two-Scale Model to the HERMES Data on Nuclear Attenuation

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Abstract

The Two-Scale Model and its improved version were used to perform the fit to the HERMES data for ν (the virtual photon energy) and z (the fraction of ν carried by hadron) dependencies of nuclear multiplicity ratios for π^+ and π^- mesons electro-produced on two nuclear targets (^{14}N and ^{84}Kr). The quantitative criterium χ^2 was used for the first time to analyse the results of the model fit to the nuclear multiplicity ratios data. The two-parameter's fit gives satisfactory agreement with the HERMES data. Best values of the parameters were then used to calculate the ν- and z - dependencies of nuclear attenuation for π^0, K$^+$, K$^-$ and $\bar{p}$ produced on ^{84}Kr target, and also make a predictions for ν, z and the Q^2 (the photon virtuality) - dependencies of nuclear attenuation data for those identified hadrons and nuclea, that will be published by HERMES.

1 Introduction

Studies of hadron production in deep inelastic semi-inclusive lepton-nucleus scattering (SIDIS) offer a possibility to investigate the quark (string, color dipole) propagation in dense nuclear matter and the space-time evolution of the hadronization process. It is well-known from QCD, that confinement forbids existence of an isolated color charge (quark, antiquark, etc.). Consequently, it is clear that after Deep Inelastic Scattering (DIS) of lepton on intra-nuclear nucleon, the complicated colorless pre-hadronic system arises. Its propagation in the nuclear environment involves processes like multiple interactions with the surrounding medium and induced gluon radiation. If the final hadron is formed inside the nucleus, the hadron can interact via the relevant hadronic cross section, causing further reduction of the hadron yield [1]. QCD at present can not describe the process of quark hadronization because of the major role of "soft" interactions. Therefore, the investigation of quark hadronization is of basic importance for development of QCD. For this purpose we investigate in this paper the Nuclear Attenuation (NA), which is the ratio of the differential multiplicity on nucleus to that on deuterium. At present there exist numerous phenomenological models for investigation of the NA problem [2]-[14]. In this work we use the Two-Scale Model [4] and its improved version to perform the fit to the HERMES NA data [15, 16]. For the fitting purposes we use the more precise part of data, including data for ν- and z - dependencies of NA of π^+ and π^- mesons on two nuclear targets (^{14}N and ^{84}Kr). The ν- and z - dependencies of NA for π^0, K$^+$, K$^-$ and antiproton, produced on

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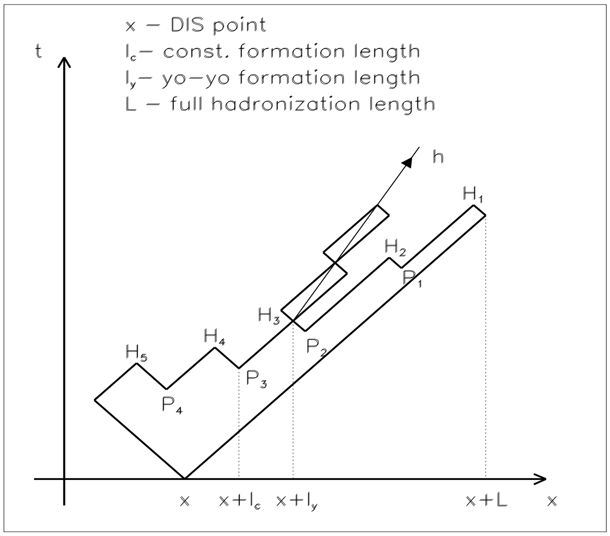

Figure 1: Space-time structure of hadronization in the string model. The two constituents of the hadron are produced at different points. The constituents of the hadron h are created at the points P_2 and P_3. They meet at H_3 to form the hadron.

^{84}Kr target we describe with best values of parameters obtained from the above mentioned fit. The best set of parameters are used also for prediction of ν, z and Q^2- dependencies of NA for the data on those identified hadrons and targets that will be published soon by HERMES [17]. The remainder of the paper is organized as follows. In section 2 we briefly remind about the Two-Scale Model. In section 3 we discuss the possibility of inclusion of the Q^2-dependence in the Two-Scale Model . In section 4 we describe the scheme we used to improve the Two-Scale Model, substituting the step-by-step increase of the string-nucleon cross section by a smooth raising function. In section 5 we present the results of the model fit to the HERMES data. Our conclusions are given in section 6.

2 The Two-Scale Model

The Two-Scale Model is a string model, which was proposed by EMC [4] and used for the description of their experimental data. Basic formula is:

$$R_A = 2\pi \int_0^\infty bdb \int_{-\infty}^\infty dx\rho(b,x)[1 - \int_x^\infty dx'\sigma^{str}(\Delta x)\rho(b,x')]^{A-1} \tag{1}$$

where b - impact parameter, x - longitudinal coordinate of the DIS point, x′- longitudinal coordinate of the string-nucleon interaction point, $\sigma^{str}(\Delta$x) - the string-nucleon cross section on distance Δx = x'-x from DIS point, $\rho(b,x)$ - nuclear density function, A - atomic mass number.

The model contains two scale (see Fig. 1): τ_c (l_c) - constituent formation time (length), and τ_h (l_h) - yo-yo formation time (length), [2] yo-yo formation means, that the colorless

[2] in relativistic units ($\hbar$ = c = 1, where $\hbar = h/2\pi$ is the Plank reduced constant and c - speed of light)

system with valence content and quantum numbers of final hadron arises, but without its "sea" partons. The simple connection exists between τ_h and τ_c

$$\tau_h - \tau_c = z\nu/\kappa, \tag{2}$$

where z= E_h/ν, E_h and ν are energies of final hadron and virtual photon correspondingly, κ - string tension (string constant). Further we will use two different expressions for τ_c. The expression for τ_c obtained for hadrons containing leading quark [18]:

$$\tau_c = (1-z)\nu/\kappa. \tag{3}$$

The expression for average value of τ_c, which was obtained in [5, 19] in framework of the standard Lund model [20]:

$$\tau_c = \int_0^\infty l dl D_c(L,z,l) / \int_0^\infty dl D_c(L,z,l), \tag{4}$$

where D_c(L, z, l) is the distribution of the constituent formation length l of hadrons carrying momentum z. This distribution is:

$$D_c(L,z,l) = L(1+C)\frac{l^C}{(l+zL)^{C+1}}(\delta(l-L+zL) + \frac{1+C}{l+zL})\theta(l)\theta(L-zL-l), \tag{5}$$

where L = ν/κ, and parameter C=0.3. The path traveling by string between DIS and interaction points is Δx = x'-x. The string-nucleon cross section is:

$$\sigma^{str}(\Delta x) = \theta(\tau_c - \Delta x)\sigma_q + \theta(\tau_h - \Delta x)\theta(\Delta x - \tau_c)\sigma_s + \theta(\Delta x - \tau_h)\sigma_h \tag{6}$$

where σ_q, σ_s and σ_h are the cross sections for interaction with nucleon of initial string, open string (which becomes one of the hadron quarks being looked at) and final hadron respectively (see Fig.2 a)).

3 Inclusion of the Q^2-dependence in Two-Scale Model.

The Two-Scale Model [4] does not contain direct Q^2-dependence and operates with the average values of cross sections:

$$\sigma_q = \sigma_q(\hat{Q}^2); \qquad \sigma_s = \sigma_s(\hat{Q}^2_{\tau_c}), \tag{7}$$

where $\hat{Q}^2$ is average value of Q^2 obtained in experiment for initial state and $\hat{Q}^2_{\tau_c}$ is the same for open string, or for time τ_c after DIS point. Obviously that $\hat{Q}^2_{\tau_c}$ is smaller than $\hat{Q}^2$.

because after DIS the string radiates gluons and diminishes its virtuality. QCD predicts the Q^2-dependence of string-nucleon cross section in the form [21, 22]:

$$\sigma_q(Q^2) \sim 1/Q^2; \qquad \sigma_s(Q^2_{\tau_c}) \sim 1/Q^2_{\tau_c}. \tag{8}$$

Using this prediction we can write the cross section for initial string as

$$\sigma_q(Q^2) = (\hat{Q}^2/Q^2)\sigma_q(\hat{Q}^2). \tag{9}$$

In the same way can be written the expression for open string cross section

$$\sigma_s(Q^2_{\tau_c}) = (\hat{Q}^2_{\tau_c}/Q^2_{\tau_c})\sigma_s(\hat{Q}^2_{\tau_c}), \tag{10}$$

where $Q^2_{\tau_c}$ is the virtuality of string for time τ_c after DIS point, and $\hat{Q}^2_{\tau_c}$ is the same for average value of Q^2. For estimation of ratio $\hat{Q}^2_{\tau_c}/Q^2_{\tau_c}$ we adopt the scheme given in Ref. [23, 24]. In according with this scheme, for the time t the quark decreases its virtuality from the initial one, Q^2, to the value $Q^2(t)$

$$Q^2(t) = \nu(t)\frac{Q^2}{\nu(t) + tQ^2}, \tag{11}$$

where $\nu(t) = \nu - \kappa t$. The calculations shown, that for HERMES kinematics ($1.2 < Q^2 < 9.5$ GeV^2 and $\hat{Q}^2 = 2.5$ GeV^2), values for ratio $\hat{Q}^2_{\tau_c}/Q^2_{\tau_c}$ are close to 1 (for τ_c in form of (3) it changes in region $0.97 \div 1.04$ and in form of (4) in region $0.92 \div 1.12$). This means that σ_s is practically constant.

4 Improved version of Two-Scale Model

In the Two-Scale Model the string-nucleon cross section is a function which jumps in points $\Delta x = \tau_c$ and τ_h. In reality the cross section increases smoothly until it reaches the size of hadronic cross section. That is why we need to improve the model in order to obtain the smooth increase of the cross section (see Fig. 2). We introduce the parameter c ($0 < c < 1$) in order to take into account the well known fact, that string starts to interact with hadronic cross section soon after creation of the first constituent quark of the final hadron, before creation of second constituent. The string-nucleon cross section starts to increase from DIS point, and reaches the value of the hadron-nucleon cross section at $\Delta x = \tau$. However, in that case one cannot deduce the exact form of σ^{str} from perturbative QCD, at least in region $\Delta x \sim \tau$. This means, that some model for the shrinkage-expansion mechanism has to be invented. We use four versions for σ^{str}. Two versions we took from Ref. [25]. The first version is based on quantum diffusion:

$$\sigma^{str}(\Delta x) = \theta(\tau - \Delta x)[\sigma_q + (\sigma_h - \sigma_q)\Delta x/\tau] + \theta(\Delta x - \tau)\sigma_h \tag{12}$$

where $\tau = \tau_c + c\Delta\tau$, $\Delta\tau = \tau_h - \tau_c$,

The second version follows from naive parton case:

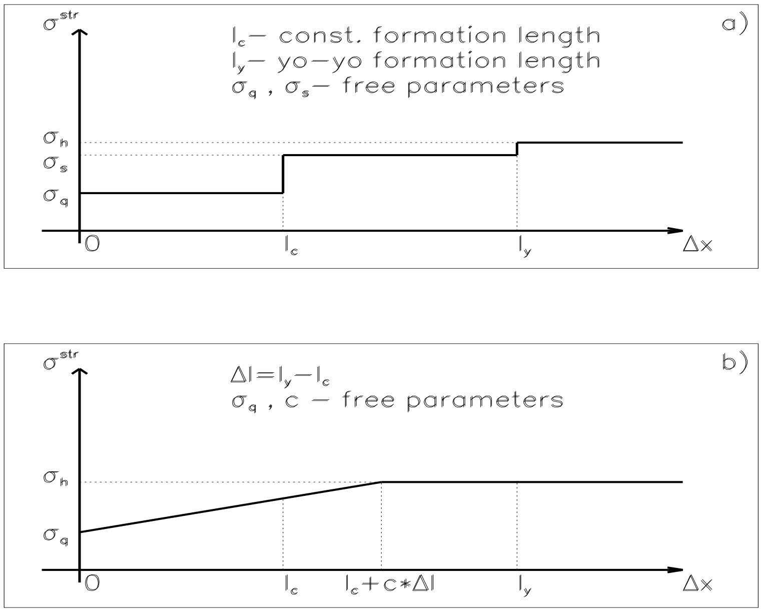

Figure 2: a) The behaviour of the string-nucleon cross section as a function of distance in the Two-Scale Model. b) The same as in a) for improved Two-Scale Model with taking into account more realistic smoothly increasing string-nucleon cross section.

$$\sigma^{str}(\Delta x) = \theta(\tau - \Delta x)[\sigma_q + (\sigma_h - \sigma_q)(\Delta x/\tau)^2] + \theta(\Delta x - \tau)\sigma_h \tag{13}$$

We used also two other expressions for σ^{str} [2, 6]:

$$\sigma^{str}(\Delta x) = \sigma_h - (\sigma_h - \sigma_q)exp(-\Delta x/\tau) \tag{14}$$

and:

$$\sigma^{str}(\Delta x) = \sigma_h - (\sigma_h - \sigma_q)exp(-(\Delta x/\tau)^2) \tag{15}$$

One can easily note that at $\Delta x/\tau \ll 1$ the expressions (14) and (15) turn into (12) and (13), correspondingly.

5 Results

In this work we have formulated one of possible improvements of the Two-Scale model and performed a fit to the HERMES data [15, 16]. Only the data for NA for ν- and z - dependencies of π^+ and π^- mesons on ^{14}N and ^{84}Kr nuclea were used for the actual fit. Furthermore, NA for ν- and z - dependencies of other hadrons produced on ^{84}Kr target were calculated. Also based on the best fit parameters one can make different predictions for ν, z and Q^2 - dependencies for those identified hadrons and nuclea, that will be published by

HERMES [17]. The string tension (string constant) was fixed at a static value determined by the Regge trajectory slope [24, 26]

$$\kappa = 1/(2\pi\alpha'_R) = 1GeV/fm \tag{16}$$

We use the following Nuclear Density Functions (NDF):

For ^{4}He and ^{14}N we use the Shell Model [27], according to which four nucleons (two protons and two neutrons), fill the s - shell, and other A-4 nucleons are on the p - shell:

$$\rho(r) = \rho_0(\frac{4}{A} + \frac{2}{3}\frac{(A-4)}{A}\frac{r^2}{r_A^2})exp(-\frac{r^2}{r_A^2}), \tag{17}$$

where r_A=1.31 fm for ^{4}He and r_A=1.67 fm for ^{14}N.

For ^{20}Ne, ^{84}Kr and ^{131}Xe we use Woods-Saxon distribution

$$\rho(r) = \rho_0/(1 + exp((r - r_A)/a)). \tag{18}$$

The three sets of NDF were used for the fitting with the following corresponding parameters:

First set (NDF=1) [28].

$$a = 0.54\ fm; \qquad r_A = (0.978 + 0.0206A^{1/3})A^{1/3}\ fm \tag{19}$$

Second set (NDF=2) [29]

$$a = 0.54\ fm; \qquad r_A = (1.19A^{1/3} - 1.61/A^{1/3})\ fm \tag{20}$$

Third set (NDF=3) [30]

$$a = 0.545\ fm; \qquad r_A = 1.14A^{1/3}\ fm. \tag{21}$$

where ρ_0 are determined from normalization condition:

$$\int d^3r\rho(r) = 1 \tag{22}$$

Parameter a is practically the same for all three sets, radius r_A for the third set is larger approximately on 6% than for second and first sets. From the fit we determined two parameters for Two-Scale Model σ_q and σ_s. In case of the improved Two-Scale Model the fitting parameters are σ_q and c.

Determination of the parameter c is represented in Section 4. For fitting we used two expressions for τ_c, which are equations (3) and (4), and five expressions for $\sigma^{str}(\Delta x)$ (6), (12)- (15). The results of the performed fit are presented in Tables 1, 2a and 2b. As we have mentioned above only part of HERMES experimental data was used for the fitting procedure, including ν - and z - dependencies of NA for π^+ and π^- on ^{14}N and ^{84}Kr nuclea. For each measured bin the information on the values of $\hat{z}$ (averaged over the given ν bin) in case of ν dependence, and $\hat{\nu}$ in case of z dependence was taken from experimental data. Use of this information allows to avoid the problem of additional integration over z and ν in formulae (1).

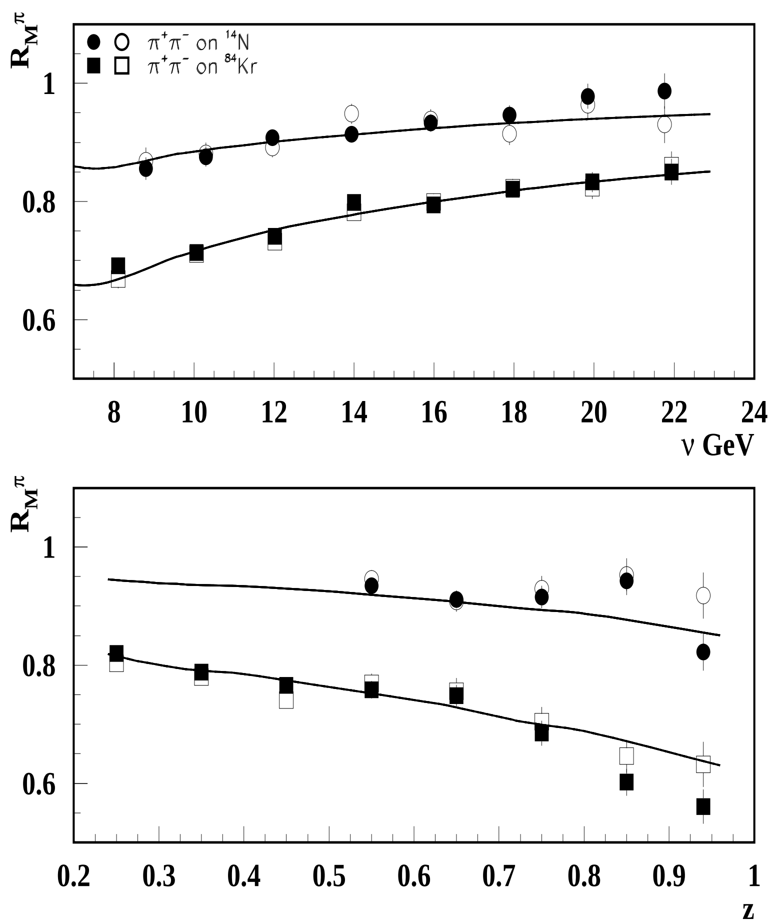

Figure 3: Hadron multiplicity ratio R of charged pions for ^{14}N and ^{84}Kr nuclea as a function of ν (upper panel), z (lower panel). The theoretical curves correspond the calculations in Improved Two-Scale Model performed with NDF (17) for ^{14}N and NDF (19) for ^{84}Kr and σ^{str} (12) with τ_c in form (3) for the values of parameters: σ_q=0.46mb, c=0.32.

In Table 1 the best values for fitted parameters, their errors and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2. N$_{exp}$ and N$_{par}$ are the numbers of experimental points and fitting parameters which were used.) for the Two-Scale Model are represented. Two different expressions for τ_c, and three different sets of parameters for NDF (^{84}Kr) were used. Tables 2a and 2b contain the best values for fitted parameters, their errors and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2) for the Improved Two-Scale Model. Four different expressions for σ^{str} were used. Only difference between Tables 2a and 2b is the form of τ_c. The results for Two-Scale Model (Table 1) are qualitatively close to the results of Ref. [4]. The values of $\sigma_q \ll \sigma_h$ and σ_s are approximately equal to σ_h. σ_q in our case is larger than the same in Ref. [4], because $\hat{Q}^2$ for HERMES kinematics is smaller than in EMC kinematics. The minimum values for χ^2/d.o.f. (best fit) were obtained for the Improved Two-Scale Model with the constituent formation time τ_c in form of (3) (see Table 2a).

The results for NA, calculated with the best values of fitting parameters for improved Two-Scale Model, for ν and z dependencies of produced charged pions on ^{14}N and ^{84}Kr targets are presented on Fig. 3.

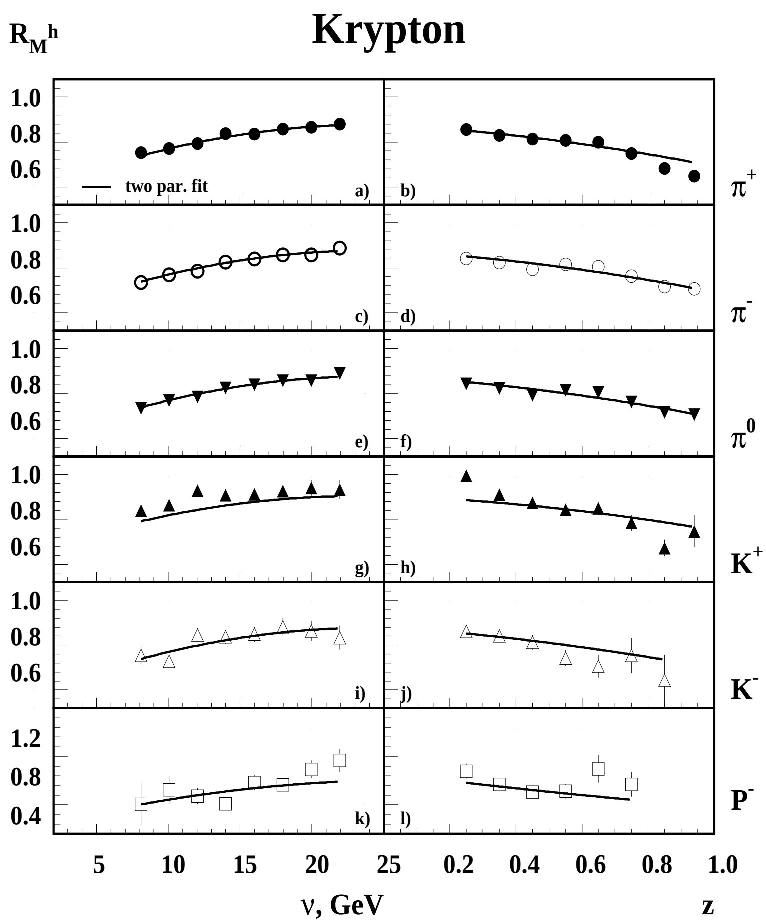

Figure 4: Hadron multiplicity ratio R of different species of hadrons produced on ^{84}Kr target [16] as a function of ν (left panel) and z (right panel). The curves are calculated with the best fit parameters described in the caption of Fig. 3.

In Fig. 4 one can see the ν and z dependencies for all identified hadrons produced on ^{84}Kr target. The values of σ_h (hadron-nucleon inelastic cross section) used in this work are equal to: $\sigma_{\pi^+} = \sigma_{\pi^-} = \sigma_{\pi^0} = \sigma_{K^-} = 20$ mb, $\sigma_{K^+} = 14$ mb and $\sigma_{\bar{p}} = 42$ mb. The curves correspond to the improved Two-Scale model with the best set of parameters.

In Fig. 5 we present the results of the Two-Scale Model and its improved version in comparison with the experimental data for NA of charged hadrons on ^{63}Cu target [4] performed in region of ν and Q^2 values higher, than in HERMES kinematics. In order to compare with the EMC data we redefined σ_q to the $\hat{Q}^2_{EMC}$, according to the expression (9).

We represent the NA ratio as a function of inverse Q^2, because of connection of this dependence with the Higher Twist effects. Indeed, from the equations (9),(6), (12)-(15) and (1) we can conclude, that in first approximation the expansion over the degrees of 1/Q^2 for NA ratio can be represented in form $R_A = a + b/Q^2$, where b is negative.

One has to note, that for calculation of 1/Q^2-dependence, the σ_q(Q^2) was used instead of σ_q. Corresponding expression is given by (9). We also take into account nuclear effects in deuterium. This means, that instead of a simple formula (1), we use for calculations the

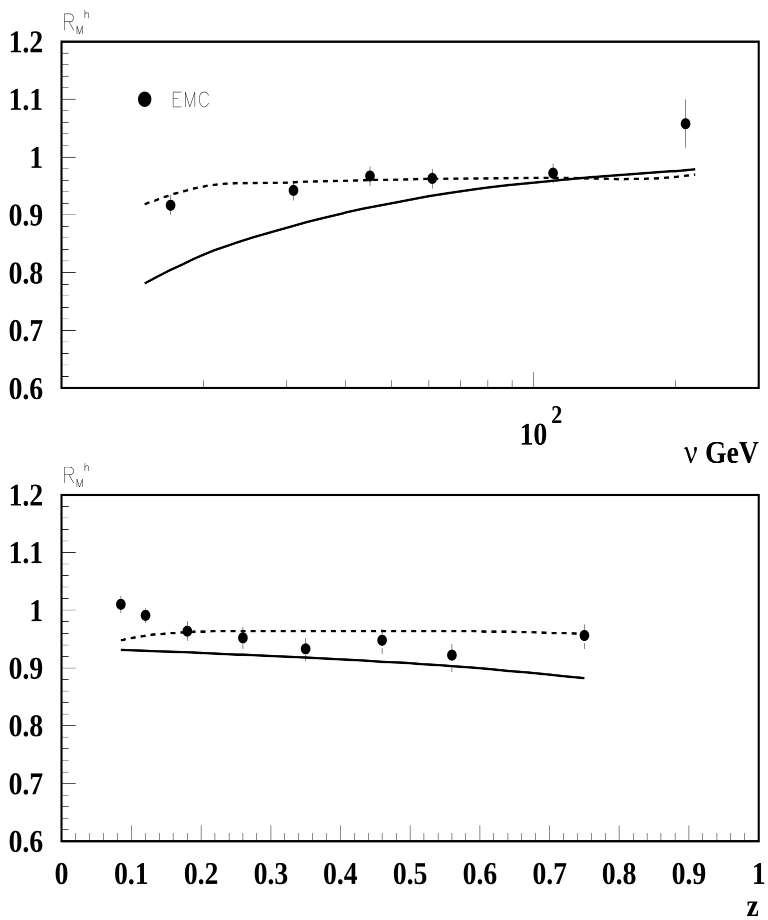

Figure 5: Hadron multiplicity ratio R of charged hadrons for ^{63}Cu nucleus as a function of ν (upper panel) and z (lower panel). The dashed curves correspond to the Two-Scale Model, the solid ones to the improved version. The solid curves are calculated with the best set of parameters described in the caption of Fig. 3. The dashed curves are calculated with $\tau_c(4)$, NDF (19), σ_q = 4.2 mb nad σ_s= 16.6 mb.

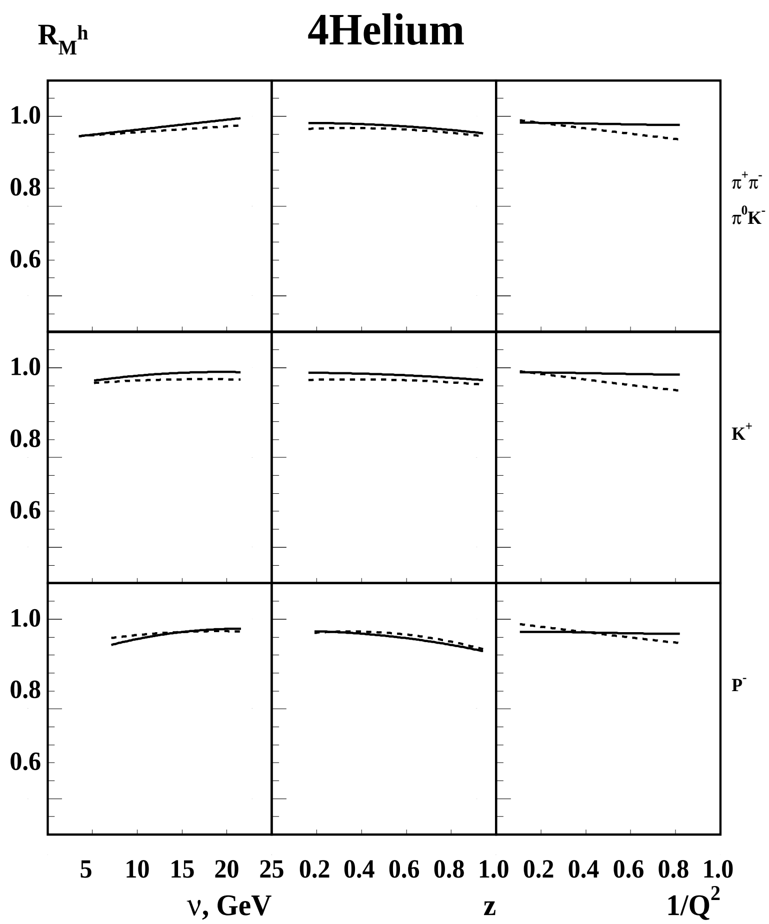

Figure 6: Hadron multiplicity ratio R of different species of hadrons produced on ^{4}He target as a function of ν (left panel), z (central panel) and Q^2 (right panel). The solid curves are the best fit using improved Two-Scale model with the parameters described in the caption of Fig. 3. The dashed curves correspond to the best fit using the simple Two-Scale model with τ_c defined in (4), NDF (17), $\sigma_q = 4.2$ mb and $\sigma_s = 16.6$ mb

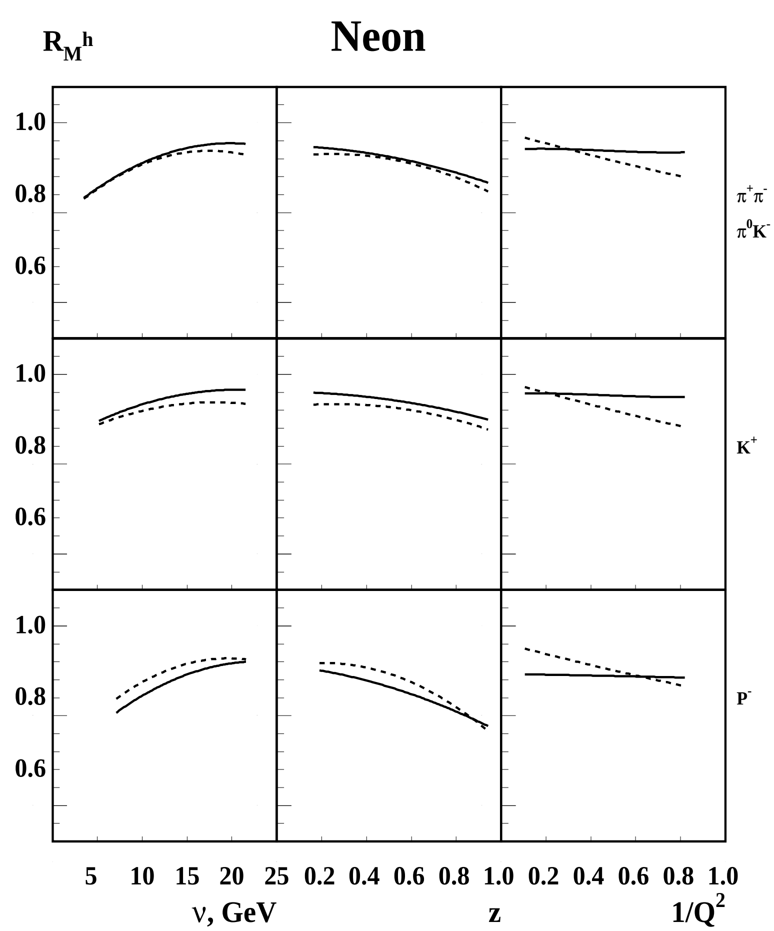

Figure 7: The same as described in the caption of the Fig. 6 done for ^{20}Ne target.

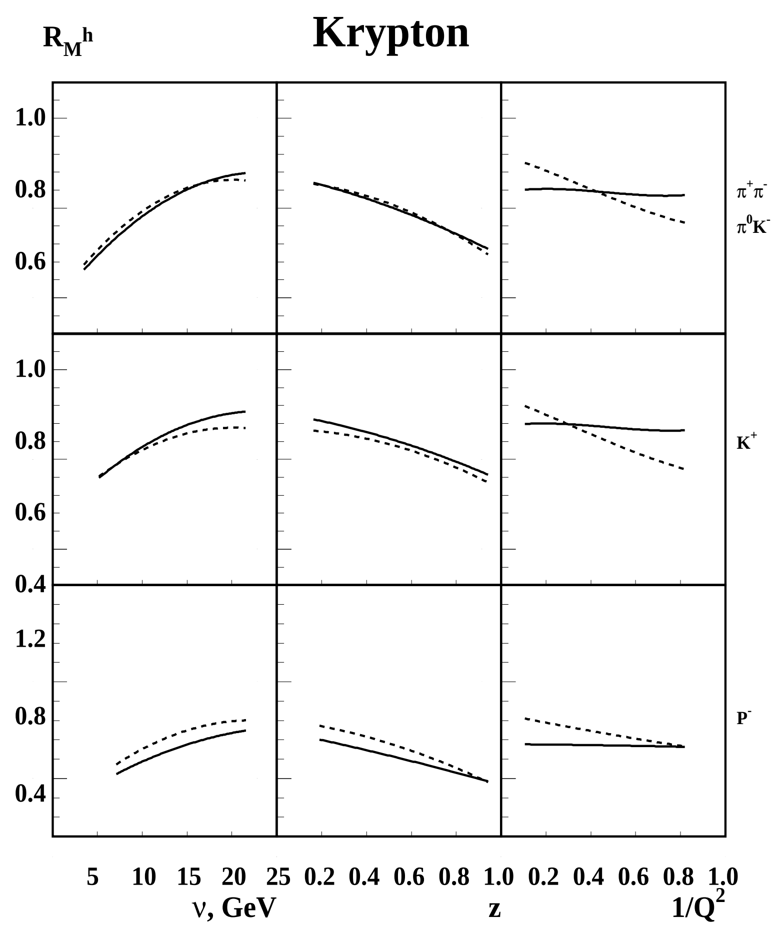

Figure 8: The same as described in the caption of the Fig. 6 done for ^{84}Kr target.

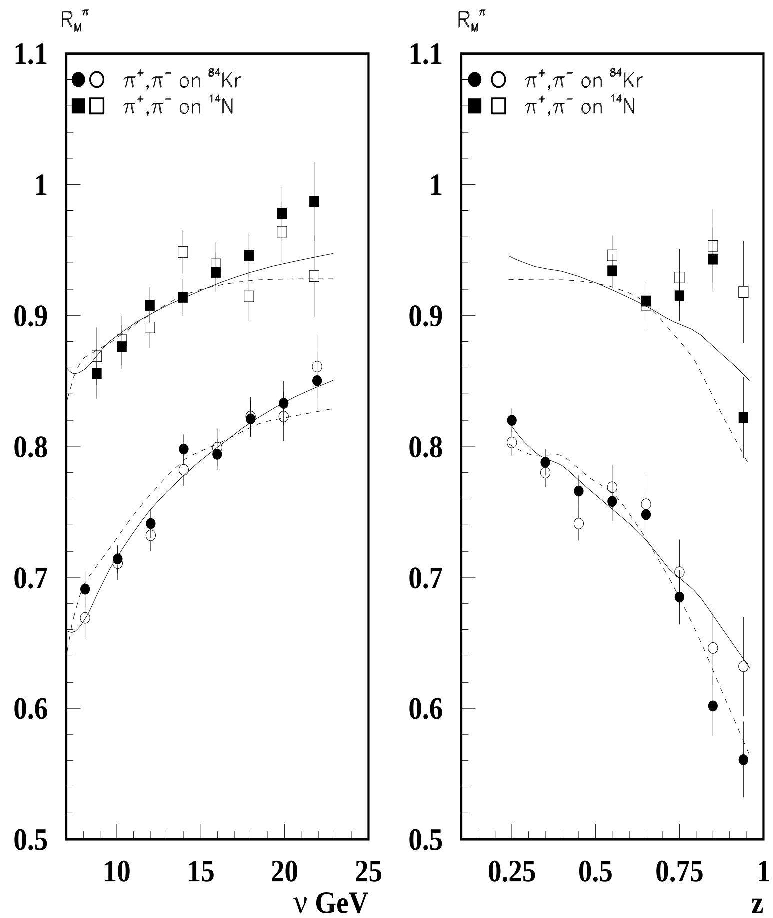

Figure 9: Descriptive ability of the Two-Scale model and its Improved version: right panel - for z, and left panel - for ν dependencies of NA. The solid lines on both panels correspond to the Improved version, the dashed ones are for simple Two-Scale model.

ratio of (1) for nucleus to the (1) for deuterium. For deuterium as NDF we use Hard Core Deuteron Wave Functions from Ref. [31].

Using the best set of parameters obtained by fitting the published HERMES data [15, 16] we calculated the predictions for the new set of the most precise in the world HERMES data [17] for ^{4}He (Fig. 6),^{20}Ne (Fig. 7) and ^{84}Kr (Fig. 8).

In order to demonstrate the achieved advantages for Imroved Two-Scale model not only on the level of obtained χ^2 values, one can compare how these two versions are describing the NA data for pions on two nuclear targets for z (see right panel of Fig. 9) and ν (see left panel of Fig. 9) dependencies. It's clearly seen from this plot that being about the same for ν dependence these two versions remarkable differ for z dependence.

The last Figure 10 is related to the predictions, done for already presented by HERMES [17] data on ^{4}He, ^{20}Ne and ^{84}Kr targets with the extended kinematics, as well as for ^{131}Xe target, on which the data is awaiting soon from the HERMES Collaboration. Two set of the best fit parameters were fixed: one marked as a dashed curves on Fig. 10 is related to the simple Two-Scale Model, next one, marked as a solid curves is related to the Improved version of the Two-Scale Model. Left panel corresponds to z dependence of

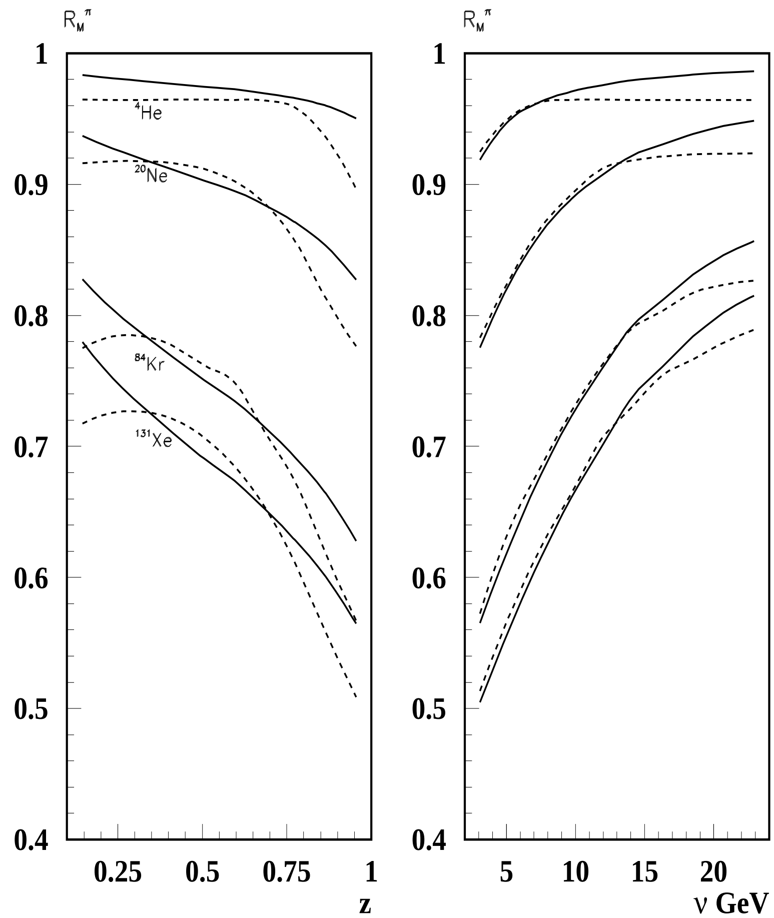

Figure 10: Two-Scale model (dashed lines) and its improved version (solid lines). The predictions for data on ^{4}He, ^{20}Ne, ^{84}Kr and ^{131}Xe done for z and ν dependencies of NA.

NA for pions, right panel is related to the ν dependence of NA for pions. We can note that again as for other nuclear targets, the difference in simple and improved versions is remarkable for Xe in z dependence.

6 Conclusions.

- The HERMES data for ν- and z - dependencies of nuclear attenuation of π^+ and π^- mesons on two nuclear targets (^{14}N and ^{84}Kr) were used to perform the fit of the Two-Scale Model and its Improved Version.

- Criterion χ^2 was used for the first time to analyse the nuclear attenuation data fit.

- Two-parameter fit demonstrates satisfactory agreement to the HERMES data. Minimum χ^2 (best fit) was obtained for improved Two-Scale Model, including expressions (12) for σ^{str} and (3) for τ_c. The published HERMES data do not give the possibility to make a choice between expressions (12)-(15), as well as to prefere definition (3) or

(4) for τ_c, because they give close values of χ^2. Preferable NDF's are set number one and two.

- More precise data expected from HERMES [17] will provide essentially definite situation with the choice of preferable NDF, expressions for σ^{str} and τ_c.
- In all versions we have obtained $\sigma_q \ll \sigma_h$. This indicates that at early stage of hadronization process Color Transparency takes place.

	$\tau_c(3)$			$\tau_c(4)$		
NDF	σ_q (mb)	σ_s (mb)	χ^2 /d.o.f.	σ_q (mb)	σ_s (mb)	χ^2 /d.o.f.
1	5.3±0.01	17.1±0.08	4.3	4.2±0.01	16.6±0.07	2.3
2	5.5±0.01	17.7±0.08	4.5	4.3±0.01	17.3±0.07	2.4
3	5.8±0.01	18.3±0.08	4.8	4.4±0.01	18.1±0.07	2.6

Table 1. The Two-Scale Model. Best values for fitted parameters and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2)

	$\sigma_{str}(12)$			$\sigma_{str}(13)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	0.46±0.02	0.32±0.03	1.4	3.5±0.01	0.23±0.002	1.9
2	0.62±0.01	0.31±0.01	1.7	3.7±0.01	0.22±0.02	2.1
3	0.78±0.02	0.30±0.03	1.8	3.9±0.01	0.21±0.003	2.3

	$\sigma_{str}(14)$			$\sigma_{str}(15)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	1.1±0.01	0.15±0.03	2.1	3.7±0.01	0.15±0.02	2.3
2	1.3±0.02	0.15±0.03	2.4	3.9±0.01	0.14±0.02	2.6
3	1.5±0.02	0.14±0.03	2.8	4.1±0.01	0.14±0.02	2.9

Table 2a. The Improved Two-Scale Model:$\tau_c(3)$. Best values for fitted parameters and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2).

	$\sigma_{str}(12)$			$\sigma_{str}(13)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	0.0±0.001	0.56±0.02	4.6	0.97±0.01	0.17±0.002	1.6
2	0.0±0.002	0.53±0.02	4.3	1.0±0.02	0.17±0.02	1.5
3	0.0±0.002	0.49±0.006	4.0	1.1±0.02	0.16±0.02	1.6

	$\sigma_{str}(14)$			$\sigma_{str}(15)$		
NDF	σ_q (mb)	c	χ^2 /d.o.f.	σ_q (mb)	c	χ^2 /d.o.f.
1	0.0±0.001	0.24±0.02	3.0	1.5±0.02	0.103±0.02	1.5
2	0.0±0.002	0.21±0.02	2.9	1.7±0.02	0.096±0.02	1.6
3	0.0±0.002	0.18±0.02	2.8	1.8±0.02	0.089±0.02	1.8

Table 2b. The Improved Two-Scale Model:$\tau_c(4)$. Best values for fitted parameters and χ^2/d.o.f. (N$_{exp}$=58, N$_{par}$=2).

We do not include in consideration NA of protons, because in this case additional mechanisms connected with color interaction (string- flip) and final hadron rescattering become essential (see for instance Ref. [3, 5])

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